Available online at www.sciencedirect.com
SCIENCE DIRECT®
ACADEMIC PRESS
ICARUS
Icarus 162 (2003) 453–459
www.elsevier.com/locate/icarus

Note

Clues to the origin of Jupiter's Trojans: the libration amplitude distribution

F. Marzari,[a],* P. Tricarico,[a] and H. Scholl[b]

[a] *Dipartimento di Fisica, University of Padua, Italy*
[b] *Observatoire de la Cote d'Azur, Nice, France*

Received 25 June 2002; revised 10 December 2002

Abstract

We model with numerical algorithms the dynamical processes that possibly lead to the trapping of Jupiter's Trojans from a primordial population of planetesimals orbiting nearby a proto-Jupiter. The predictions of models based on mutual planetesimal collisions and on the mass growth of Jupiter are compared with observations. In particular, we concentrate on the distribution of the libration amplitude. The two mechanisms for trapping reproduce closely the libration amplitude distribution of the real Trojans only when the long-term dynamical diffusion described by Levison et al. (1997, Nature 385, 42–44) is taken into account.

Keywords: Celestial mechanics; Asteroids

1. Introduction

Theoretical studies on jovian Trojans mostly concentrate on two subjects, their origin and their long-term stability. It is obvious that there is an interrelation between these two topics since what we observe at present is the primordial population slowly eroded over the Solar System age by dynamical instability (Levison et al., 1997) and collisions (Marzari et al., 1997). Any cosmogonical model which tries to explain the origin of Trojans must in some way account for the slow outflow from the two swarms. Otherwise, its predictions cannot be compared to observations.

Most scenarios assume that Trojans originated during the early phase of Jupiter's formation when a large number of planetesimals were roaming around near the growing planet in a highly perturbed environment. Mutual collisions, changes in the gravity field due to the planet's mass increase, and, for the smaller planetesimals, gas drag affected the trajectories of the planetesimals and can have placed them into stable tadpole orbits. Shoemaker et al. (1989) conjectured that mutual collisions between planetesimals could inject a significant amount of bodies into Trojan orbits. Their idea has never been tested numerically, but it sounded reasonable since the sudden changes of orbital elements caused by collisions were randomly distributed and could have resulted in capture. Marzari and Scholl (1998a, 1998b) and Fleming and Hamilton (2000) simulated the process of Trojan trapping by the rapid mass growth of Jupiter. They found a high capture rate, even too efficient when compared to the present estimated mass in the two Trojan swarms (Jewitt et al., 2000). Peale (1993) considered the orbital dissipation caused by gas drag and showed that small planetesimals, with diameters between 5 and 10 km, can be stabilized as Trojans. He assumed that the presently observed large Trojans with diameters exceeding 100 km formed by accretion of the smaller planetesimals captured by gas drag.

All of the above-mentioned mechanisms likely worked in synergy to produce the presently observed Trojan swarms of Jupiter. In this paper we investigate if two of the trapping mechanisms, one based on mutual collisions and the other related to the mass growth of the planet, can reproduce the

* Corresponding author.
E-mail address: marzari@pd.infn.it (F. Marzari).

0019-1035/03/$ – see front matter © 2003 Elsevier Science (USA). All rights reserved.
doi:10.1016/S0019-1035(03)00026-5

observed orbital distribution of Jupiter's Trojans, in particular their libration amplitude distribution. We do not include nebular gas drag effects in our simulations since it affects mainly small bodies and, at present, we do not have detailed data on the orbital distribution of smaller Trojans. To investigate the predictions of the two models for Trojan capture we adopt a numerical model that includes both the effects of mutual collisions between planetesimals and of Jupiter's mass growth. We derive the orbital distribution of the final population of planetesimals trapped as Trojans and, concentrating in particular on the libration amplitude distribution, we compare the model's predictions with observations. Good agreement is obtained in particular when the slow dynamical outflow of bodies from the Trojan swarms (Levison et al., 1997) is taken into account.

2. The numerical algorithm

The numerical code used to simulate the evolution of planetesimals into Trojans includes both the effects of the mass growth of Jupiter and of the mutual collisions between planetesimals. The dynamical system we adopt is a four-body model that includes Jupiter and Saturn. The planetesimals are treated as massless bodies and their mutual gravitational interactions are neglected. Both the planets may have increasing masses and their initial orbital elements, when their mass growth is considered, are chosen in order to reproduce closely, at the end of the simulation, the present secular frequencies (Marzari and Scholl, 1998a, 1998b). The initial orbital elements of the planetesimals are selected randomly within an annulus surrounding the orbit of the planet. We do not consider in our simulations a possible migration of the planets that, anyway, should not affect significantly our results. Fleming and Hamilton (2000) have shown that even a change of 1 AU does not affect significantly the librational amplitude. Michtchenko et al. (2001) studied the effects of resonance crossing during the migration phase and concluded that a sufficiently small and fast orbital drift would not appreciably change the distribution of known Trojans.

The algorithm to compute when a collision occurs during the numerical integration of the planetesimal orbits and the outcome of the collisions is described in detail in Marzari and Scholl (2000). Here we briefly summarize the main features of the algorithm. To model the collisional evolution of a system populated by hundreds of millions of planetesimals computing the orbits of only a few thousand representative bodies we use the inflated diameter approach (Charnoz et al., 2001; Thébault and Brahic, 1998) which increases artificially the impact rate. The simulations are three-dimensional and the model includes 5000 initial massless bodies in a ring surrounding the orbit of Jupiter. At each timestep, the algorithm that checks for collisions must find all the pairs of bodies that have a mutual distance smaller than twice the inflated radius. For each pair that satisfies this condition a collision is assumed to occur and new orbital elements are computed for the two bodies in the approximation of inelastic collision. An important aspect of this algorithm is that the heliocentric position vector of each body involved in a collision is not changed, only the velocity is updated. The use of inflated diameters does not cause a false trapping of bodies in Trojan-type orbits.

The most time consuming part of the algorithm is to compute at each timestep the mutual distances between bodies to find potential partners for a collision. A "systolic" algorithm is used (Marzari and Scholl, 2000), where the position vectors of all planetesimals are first sorted with respect to one component, i.e., their x-component. Collisional partners of an ith planetesimal can be found only among planetesimals j that have a distance $|x_i - x_j|$ lower than the inflated diameter. Among this subgroup of bodies the other two coordinates are subsequently tested for collision. This algorithm allows a significant reduction in the number of mutual distance computations.

The inelastic collision outcome is computed following Hertzsch et al. (1997), where the relative velocity is changed in both the normal and tangential directions by two reconstitution coefficients $\epsilon_N = -0.3$ and $\epsilon_T = -1$. The third component along the perpendicular axis is left unchanged, since most of the orbits have very low inclinations with respect to the Jupiter orbit.

When the mass growth option is activated in the code, exponential mass growth for both Jupiter and Saturn is adopted following Marzari and Scholl (1998a, 1998b).

3. The distribution of the libration amplitude

According to the generally accepted standard model for the formation of Jupiter (Pollack et al., 1996) the capture of Jupiter's Trojans may have occurred during two distinct phases characterized by two different growth rates for Jupiter. In the first stage, a proto-Jupiter is slowly forming by planetesimal accretion reaching at the end, possibly on timescales on the order of some millions of years, a few Earth masses (Pollack et al., 1996). In a second phase, when proto-Jupiter has reached a critical mass, the surrounding gas of the nebula is attracted and falls on the planet probably between 10^3 and 10^5 years, a much shorter timescale than that for the accretion phase.

Planetesimals orbiting in the proximity of the growing Jupiter, after a collision close to a Lagrangian point L4 or L5 of the proto-planet can be trapped as Trojans because of the change in the orbital velocity. This is the mechanism proposed by Shoemaker et al. (1989). It works during the whole period of Jupiter's growth and it becomes more efficient when the planet grows larger since the trapping region widens. At the end of Jupiter's growth, the population of trapped Trojans have a large range of libration amplitudes. Collisions of already trapped Trojans with other Trojans or with nearby planetesimals would result either in

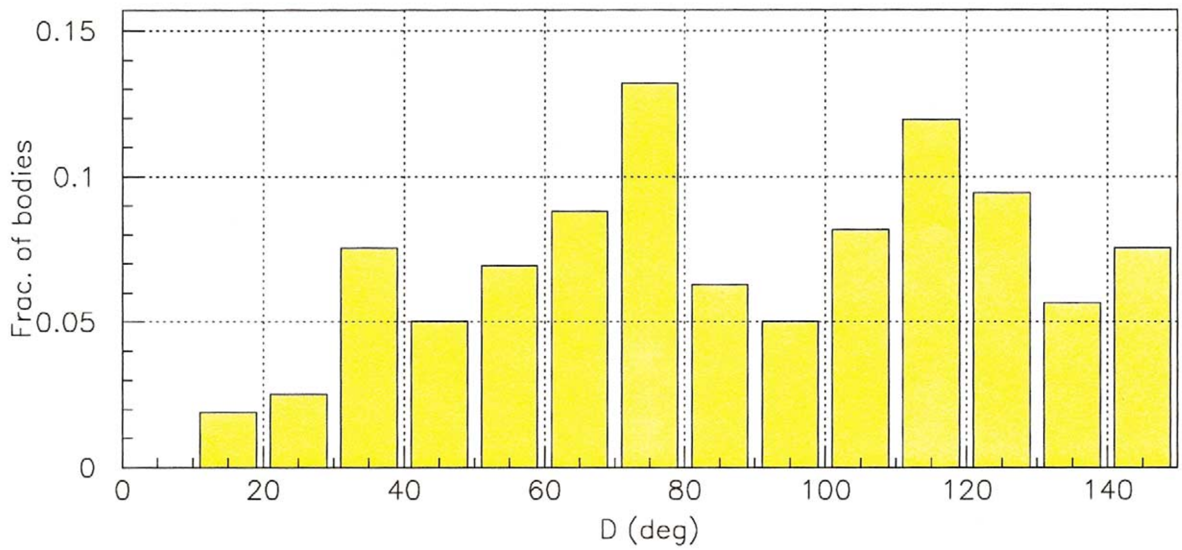

Fig. 1. Histogram of the libration amplitude distribution of planetesimals captured as Trojans by mutual collisions.

an ejection out of the Trojan cloud or in a further randomization of libration amplitudes.

We first simulated the capture of Trojans by collisions. The orbits of 5000 test bodies were integrated over 2×10^4 years with an inflated diameter tuned to give about one collision per body every 1.5×10^4 years. This timescale was mostly chosen to grant a reasonable amount of collisions during the integration timespan that could be achieved within 1 month of CPU time. It is longer than both the libration period of Trojan orbits, that is, about 1×10^3, when the proto-Jupiter is set to 10 Earth masses, and the period of libration of horseshoe orbits, that is, about 3×10^3, at a maximum in our orbital sample. For a planetesimal disk with a superficial density $\sigma = 10$ g/cm^3 and a bulk density for the bodies $\rho = 1.4$ g/cm^3 (Pollack et al., 1996) the collision rate we have adopted corresponds to the frequency with which a planetesimal of 20 km in radius hits a planetesimal of 50 km. We have to take into account that the collisions with smaller bodies are dominant and can cause a change in the orbital elements of a larger body sufficient to move the body into a Trojan-type orbit.

The initial semimajor axes of the bodies in the sample range from 4.9 to 5.5 AU, the eccentricities are between 0.0 and 0.1, and the inclinations are between 0.0° and 1.5°. They form an annulus that extends on either side of Jupiter's orbit. Both Jupiter and Saturn are included in the simulation with a fixed mass equal to 10 Earth masses. We start the simulation with 5000 bodies and after 2×10^4 years only 1803 are left. About 9% of the survivors have been trapped as Trojans and, out of these, only 12% have a libration amplitude lower than 40°. Jupiter is very effective in scattering away bodies, more than half of the initial bodies have been scattered away during the 2×10^4 years of integration. It would not be useful to continue the integration at this stage, unless we decide to add new bodies to the initial sample to model the injection of new planetesimals from nearby zones because of mutual planetesimal collisions and gas drag orbital decay. We performed two different simulations and the final distributions of the libration amplitude were statistically consistent. As a consequence, longer integration timespans would not lead to significantly different distributions of the Trojan orbital parameters.

The normalized histogram of the libration amplitude distribution of the planetesimals trapped as Trojans is shown in Fig. 1. To estimate libration amplitudes that can be considered as "proper" we integrate the survivors for additional 10^5 years within the same four-body model but without collisions. We compute the libration amplitude as a mean value of the maximum libration amplitude over running windows of 10^4 years. Some of the trapped Trojans become unstable during this additional integration and they are not considered in the final distribution. The histogram in Fig. 1 cannot be directly compared to the observed distribution of real Trojans since in Fig. 1 we have libration amplitudes relative to a 10 Earth mass Jupiter. A further step has to be considered before the comparison to the observed distribution: the growth of Jupiter to its final mass. Before modeling this stage of the Trojan history let us discuss in more detail the collisional algorithm. Does a model where collisions are treated as inelastic properly describe the effect of real impacts on the dynamics of a planetesimal swarm? Collisions cause a change in the orbital velocity of the body modifying its trajectory when close to the Lagrangian points: this can lead to capture as a Trojan. The real important point is: it does not matter in which direction the velocity "kick" is given, the important fact is that a "kick" is given. As a test, we performed a simulation where positive values were given to the constant ϵ_N and ϵ_T. The final

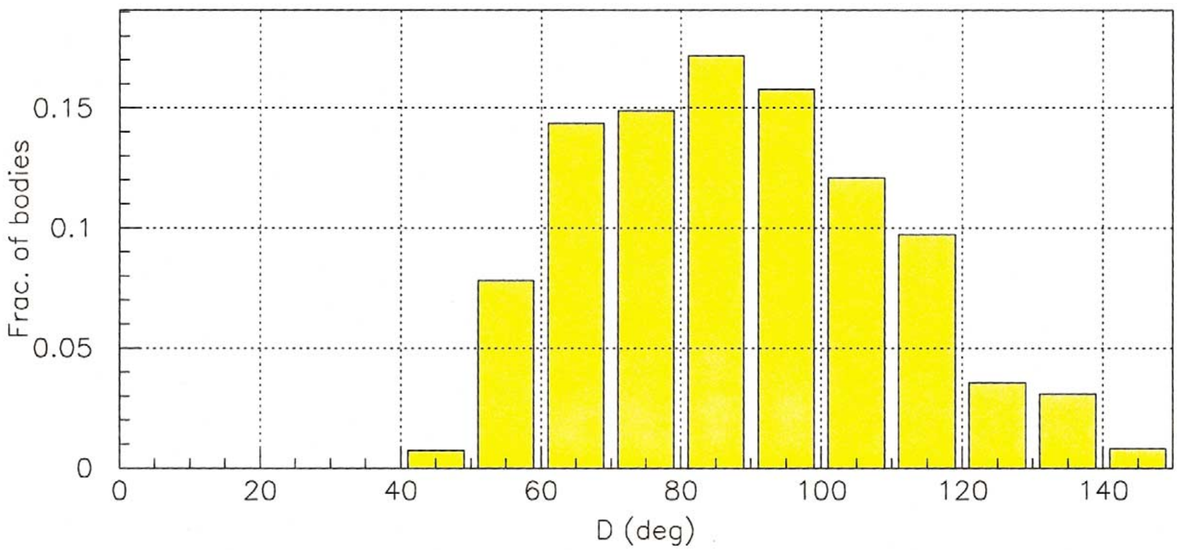

Fig. 2. Distribution of the libration amplitudes of Trojans trapped by the mass growth of Jupiter.

distribution of the Trojan libration amplitudes was very similar to that obtained with the original values of ϵ_N and ϵ_T. In conclusion, even if we cannot model collisions with fragmentation, cratering, or accretion, our algorithm is anyway able to reproduce the major effects of collisions on the process of planetesimal capture as Trojans.

At the beginning of the second phase of Jupiter growth, the gas infall is slow. It accelerates toward the end due to the larger mass of the planet. In this phase, the libration regions around L4 and L5 expand rapidly capturing a large amount of planetesimals in Trojan-type orbits (Marzari and Scholl, 1998a, 1998b; Fleming and Hamilton, 2000). We performed a numerical simulation without collisions where both Jupiter and Saturn grow at the same rate from an initial mass of 10 Earth masses to their full size on a timescale of 2×10^4 years. The choice of this timescale is somewhat arbitrary but it does not influence the final outcome of the simulation. Marzari and Scholl (1998a, 1998b) and Fleming and Hamilton (2000) have shown that the trapping process does not significantly depend on the growth timescale of the planets when this is comprised between 10^3 and 10^5 years. The distribution of the "proper" libration amplitudes, computed as above after an additional integration of the orbits for 10^5 years including Jupiter and Saturn with their present masses, is presented in Fig. 2. It shows a peak at about 80° falling off to 40°. About 37% of the initial 5000 bodies have been trapped as Trojans at the end of the simulation but, in absence of other mechanisms that modify the orbital parameters of these Trojan precursors, most of them would escape due to dynamical instability (Levison et al., 1997) within some hundred millions of years. Only a few that had a libration amplitude around 40° and low proper eccentricity would have possibly survived until the present. Moreover, if we compare the distribution in Fig. 2 with the real distribution of Jupiter's Trojans shown in Fig. 3, obtained from the proper libration amplitudes of real Trojans kindly provided by Beaugé (Beaugé and Roig, 2001), we conclude that the mass growth mechanism by itself cannot explain the present orbital distribution of Trojan swarms, unless an additional mechanism is invoked to generate low libration orbits.

At this point, what intuition suggests is that the two physical mechanisms, collisions and mass growth of Jupiter, possibly worked in synergy to produce the observed population of Trojans. We performed a two-stage simulation: in the first 2×10^4 years a fixed mass protoplanet of 10 $M_\oplus$ is embedded in a ring of 5000 planetesimals that collide with a frequency of one collision every 1.5×10^4 years. At the end of this phase, we restart the computer model but with the mass of the proto-planet growing at an exponential rate to the present mass of Jupiter on a timescale of 2×10^4 years: collisions between the planetesimals can still occur in this phase. In Fig. 4 we show the normalized histogram of the final population: 346 are trapped as Trojans (many planetesimals are dispersed during the initial proto-planet phase) and 67 have $D \leq 40°$. Why do we have more Trojans with small amplitudes D in the numerical model where collisions and mass growth are combined? First of all, the planetesimals that became Trojans because of collisions during the initial stage with a fixed-mass protoplanet, experience in the period of fast mass growth of the planet a reduction of their libration amplitudes by about 40% according to Marzari and Scholl (1998b) and Fleming and Hamilton (2000). In the histogram of Fig. 4 we mark the fraction of bodies in each libration amplitude bin that were Trojans at the end of the collisional phase and whose libration amplitude was reduced by the mass growth mechanism. Moreover, collisions continue to occur also when the mass

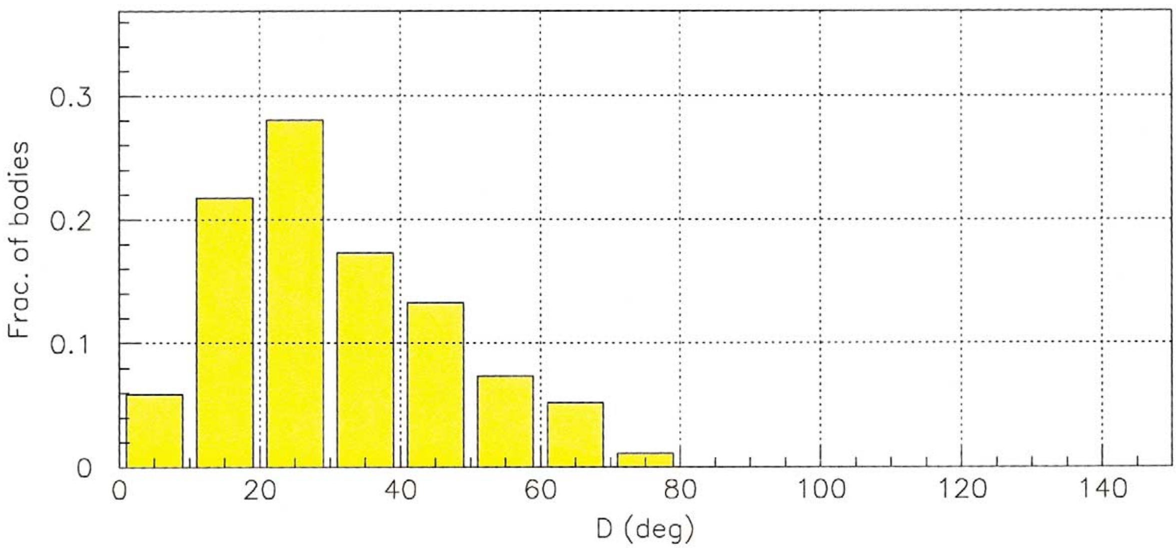

Fig. 3. Histogram showing the libration amplitude distribution of real Trojans with inclination $i < 10°$. Data have been kindly provided by Beaugé (Beaugé and Roig, 2001).

of the planet increases. The complex interplay between the expansion of the libration regions while Jupiter grows and the sudden changes of the orbital elements due to collisions inject some planetesimals, not Trojans at the end of the collisional phase, into lower libration amplitude orbits.

After a close inspection of Fig. 4 we must conclude that the comparison with Fig. 2 of real Trojans is not yet satisfactory. The peak of the distribution in Fig. 4 is around 70° and many Trojans have large libration amplitudes. A piece of the puzzle is missing: it is the dynamical outflow of those Trojans that were captured outside the stability region defined by Levison et al. (1997). To account for this additional effect in the outcome of our simulation we computed for each Trojan orbit in Fig. 4 an approximate value of the proper eccentricity by applying frequency map analysis (Laskar 1993a, 1993b; Nesvorný and Ferraz-Mello, 1997; Marzari et al., 2002) to the orbital elements computed in the subsequent integration for 10^5 years. Those bodies with proper elements outside the stability region of Levison et al. (1997) are then removed from the sample leaving us with 122 possibly primordial Trojans. It is interesting to note that the trapping mechanisms produced a population of Trojans with a uniform distribution in proper eccentricity between 0.0 and 0.15 for any value of libration amplitude. Only

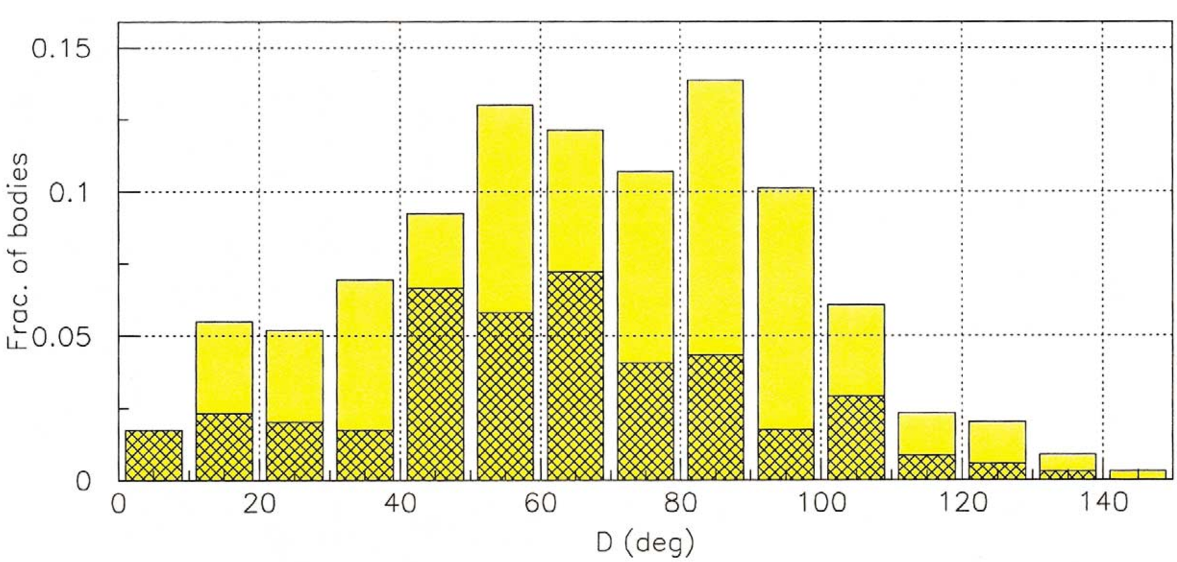

Fig. 4. Histogram of the libration amplitudes resulting from the simulation with collisions and mass growth. The darker portion of the histogram gives the fraction of bodies trapped as Trojans before the growth of the planet when the proto-Jupiter has a mass equal to 10 Earth masses.

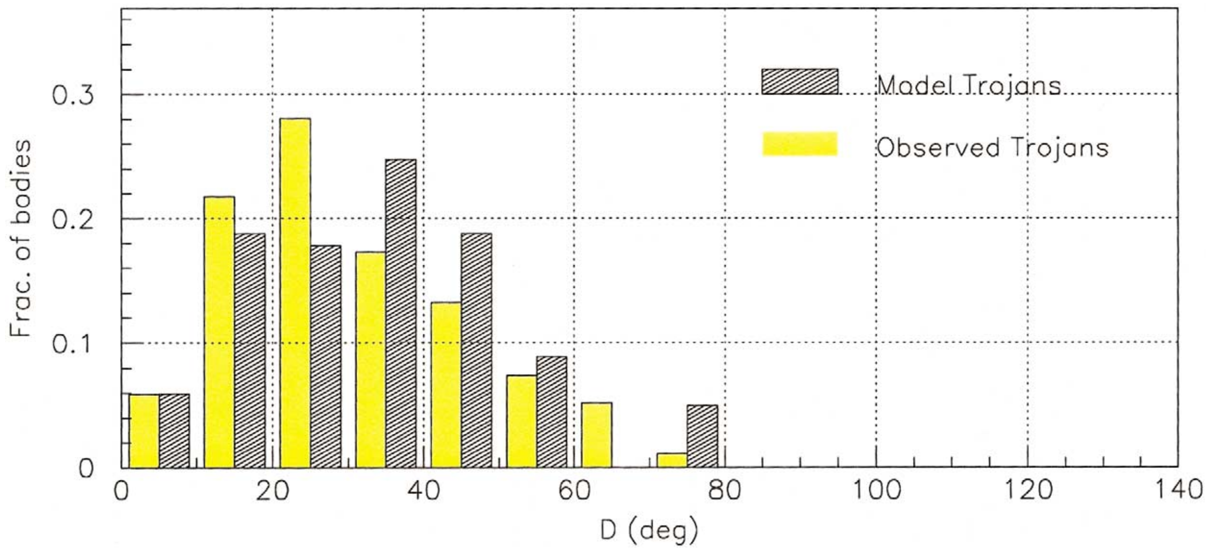

Fig. 5. Comparison of the real distribution of the libration amplitudes as from Fig. 2 with the model distribution of Fig. 4, where dynamical instability has been taken into account. The bars are shifted with respect to the center of the libration amplitude bin for comparison.

fewer bodies are trapped with eccentricities between 0.15 and 0.2. The new normalized distribution of the sample obtained after the removal of the unstable bodies is shown in Fig. 5 and it is compared with the real distribution. The agreement between the two populations in terms of libration amplitude is noticeable.

4. Discussion

Capture of planetesimals around the triangular Lagrangian points of Jupiter by mutual collisions and mass growth of the planet provide a viable scenario for the origin of Trojans. Their observed libration amplitude distribution is well reproduced by the models when also the effects of the dynamical outflow described in Levison et al. (1997) are included. We do not consider in our simulations the contribution of the collisional evolution during the whole Solar System age (Marzari et al., 1997) that might further shape the orbital distribution of Trojans. However, mutual Trojan collisions within the swarms should affect bodies of any libration amplitude and alter only marginally the original libration amplitude distribution. Our model does not explain the strong excitation in inclination of the present Trojans. Additional mechanisms such as the temporary capture of planetary embryos may have caused such excitation and might also have affected the libration amplitude distribution. This scenario needs to be investigated.

It is difficult to estimate the efficiency of the trapping by collisions during the growth of the Jupiter core. It strongly depends on the balance between the width of the Trojan regions (that is a function of the core mass), the amount of planetesimals scattered away by the growing planet, and the replenishment mechanisms as gas drag drift and collisional injection. It is also possible that most of the trappings occured only by collisions in the first phase and that the mass growth mechanism, active during the gas infall on the planet's core, contributed only by reducing the libration amplitude of the already captured Trojans. Even if this were the case, the dynamical instability would have contributed to the shaping of Trojan clouds, but its contribution might have been less relevant since almost all the Trojans would have had their libration amplitudes already reduced by the mass growth mechanism.

Acknowledgments

The authors thank C. Beaugé and A. Morbidelli for their constructive comments.

References

Beaugé, C., Roig, F., 2001. A semianalytical model for the motion of the Trojan asteroids: proper elements and families. Icarus 153, 391–415.
Charnoz, S., Thébault, P., Brahic, A., 2001. Short-term collisional evolution of a disc perturbed by a giant-planet embryo. Astron. Astrophys. 373, 683–701.
Fleming, H.J., Hamilton, D.P., 2000. On the origin of the Trojan asteroids: effects of Jupiter's mass accretion and radial migration. Icarus 148, 479–493.
Hertzsch, J.-M., Scholl, H., Spahn, F., Katzorke, I., 1997. Simulation of collisions in planetary rings. Astron. Astrophys. 320, 319–324.
Jewitt, D.C., Trujillo, C.A., Luu, J.X., 2000. Population and size distribution of small jovian Trojan asteroids. Astron. J. 120, 1140–1147.
Laskar, J., 1993a. Frequency analysis for multi-dimensional systems. Global dynamics and diffusion. Physica D 67, 257–281.
Laskar, J., 1993b. Frequency analysis of a dynamical system Celest. Mech. Dynam. Astron. 56, 191–196.
Levison, H., Shoemaker, E. M., Shoemaker, C.S., 1997. The dispersal of the Trojan asteroid swarm. Nature 385, 42–44.

Marzari, F., Scholl, H., 1998a. Capture of Trojans by a growing proto-Jupiter. Icarus 131, 41–51.
Marzari, F., Scholl, H., 1998b. The growth of Jupiter and Saturn and the capture of Trojans. Astron. Astrophys. 339, 278–285.
Marzari, F., Scholl, H., 2000. Planetesimal accretion in binary systems. Astrophys. J. 543, 328–339.
Marzari, F., Farinella, P., Davis, D.R., Scholl, H., Campo Bagatin, A., 1997. Collisional evolution of Trojan asteroids. Icarus 125, 39–49.
Marzari, F., Tricarico, P., Scholl, H., 2002. Saturn Trojans: stability regions in the phase space. Astrophys. J. 579, 905–913.
Michtchenko, T.A., Beaugé, C., Roig, F., 2001. Planetary migration and the effects of mean motion resonances on Jupiter's Trojan asteroids. Astron. J. 122, 3485–3491.
Nesvorný, D., Ferraz-Mello, S., 1997. On the asteroidal population of the first-order jovian resonances. Icarus 130, 247–258.
Peale, S.J., 1993. The effect of the nebula on the Trojan precursors. Icarus 106, 308–322.
Pollack, J.B., Hubickyj, O., Bodenheimer, P., Lissauer, J.J., Podolak, M., Greenzweig, Y., 1996. Formation of the giant planets by concurrent accretion of solids and gas. Icarus 124, 62–85.
Shoemaker, E.M., Shoemaker, C.S., Wolfe, R.F., 1989. Trojan asteroids: populations, dynamical structure and origin of the L4 and L_5 swarms, in: Binzel, R.P., Gehrels, T., Matthews, M.S. (Eds.), Asteroids II, Univ. of Arizona Press, Tucson, pp. 487–523.
Thébault, P., Brahic, A., 1998. Dynamical influence of a proto-Jupiter on a disc of colliding planetesimals. Planet. Space Sci. 47, 233–243.