

THE INSTABILITY OF VENUS TROJANS

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ABSTRACT

Using Laskar's frequency map analysis, we investigate the long-term stability of Venus Trojan orbits. By measuring the diffusion rate of proper frequencies, we outline in phase space the most stable region. It is located between 5° and 10° in proper inclination with libration amplitudes ranging from 50° to 110° . The proper eccentricity is lower than 0.13. The stable region is surrounded by secular resonances that destabilize orbits on short timescales. The dynamical half-life of orbits within the stable region is about 6×10^8 yr. The Yarkovsky effect further reduces the dynamical lifetime. Therefore, primordial Venus Trojans, if any, cannot have survived until the present. Transient Trojans, on the other hand, cannot be excluded.

Key words: celestial mechanics — minor planets, asteroids — solar system: general

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1. INTRODUCTION

The first asteroids found to librate around Jupiter's Lagrangian points L4 and L5 on tadpole orbits were named by their discoverers at the observatory in Heidelberg for mythic persons mentioned in Greek narratives, in particular Homer's *Iliad*, about the Trojan War. It became a custom to call a body librating around one of the Lagrangian points L4 or L5 of any planet a Trojan. Jupiter has the largest Trojan population, with hundreds of members, while only two Neptune and four Mars Trojans are known. No Trojans are known for the other planets.

According to the classical scenario, Jupiter Trojans accreted very near the orbit of the growing proto-Jupiter and were locked onto tadpole orbits after the full formation of Jupiter as a giant planet. The possible capture of Jupiter Trojans originating near Jupiter was shown, for instance, by Marzari & Scholl (1998) and Fleming & Hamilton (2000). However, the large orbital inclinations of Jupiter Trojans require additional dynamical mechanisms in these models. Morbidelli et al. (2005) presented a different scenario for the origin of the Jupiter Trojans based on migration of the outer planets driven by planetesimal encounters. Planetesimals originating far outside of Jupiter's orbit are transported by close encounters with the outer planets toward Jupiter's orbit. As a consequence, the three outer planets migrate outward, while Jupiter migrates slightly toward the Sun. During migration, Jupiter and Saturn cross the 2:1 orbital resonance. Planetesimals that enter Jupiter's Trojan region during the resonance crossing remain captured as Jupiter Trojans after. This model easily explains the high orbital inclinations of Jupiter Trojans. In this new scenario, the large majority of Trojans forms much farther away from Jupiter's orbit than in the classical scenario, and the capture of Trojans occurs later during migration. The absence of Trojans of a planet, or an underpopulation, as in the case of Neptune Trojans, might be due to a failure of capturing small bodies during migration.

In the case of Mars Trojans, they might have been captured at the end of the formation of the planet, when it was wandering because of close encounters with planetesimals. Even mutual collisions between planetesimals orbiting close to the Lagrangian points might have injected some fragments into tadpole orbits. Two of the four Trojans are indeed highly differentiated bodies (Rivkin et al. 2003), which means that they are collisional fragments of a large parent body. All four Mars Trojans reside in a long-term stable region (Scholl et al. 2005) confined to orbital inclinations between 15° and 30° . At lower and higher inclinations, secular resonances destabilize Trojans on gigayear timescales, according to Brassier & Lehto (2002) and Scholl et al. (2005). Presumably, these Trojans reached their present large inclinations before capture.

No Venus Trojans are currently known. Their discovery represents a challenge for observers due to the small solar elongations. This could be the reason why they are still missing. Several asteroids are known to be very near Venus co-orbitals. There are nine asteroids with semimajor axes deviating less than 0.01 AU from the semimajor axis of Venus. They all have eccentricities larger than 0.38 and, therefore, aphelion distances larger than 1 AU, which favor their discovery. Encounters with the Earth can be expected to transform their orbits temporarily into Venus co-orbitals. They may become temporary satellites or librate on horseshoe orbits or tadpole orbits. Two asteroids, 2001 CK32 and 2002 VE68, are currently known to be Venus co-orbitals (Brasser et al. 2004; Mikkola et al. 2004). The asteroid 2001 CK32 is on a horseshoe orbit, while the mean longitude of 2002 VE68 librates around 0° in a frame rotating with Venus. This asteroid can be considered a temporary satellite of Venus. Christou (2000) predicts that the asteroid 1989 VA will become a Venus co-orbital. Integrating the orbits of the nine asteroids over 100,000 yr, we found several temporary Venus co-orbitals and, in particular, three temporary Trojans. The asteroid 2001 CK32 librates for about 10,000 yr in the L5 region. The

asteroid 2003 FY6 resides a few thousand years in the L5 region, while the asteroid 2002 LT24 spends about 1000 yr in the L4 region. One cannot, therefore, exclude that there are currently undiscovered temporary Trojans on eccentric orbits. These Trojans are, however, transitional objects with short dynamical lifetimes. Can we expect to discover dynamically long-lived original Trojans with lifetimes of the order of gigayears?

The long-term stability has not yet been investigated due to the large amount of necessary computing time. Integrating the orbits of the inner planets needs about 100 times more time steps than for the outer planets. In the past, the longest numerical integrations were performed over a few megayear timescale by Mikkola & Innanen (1992). Later, Tabachnik & Evans (2000) extended the time span of integration to 100 Myr. Both author teams found stable Venus Trojans over the corresponding time-scales. Tabachnik & Evans restricted the stable region with respect to orbital inclinations to below 16° . An explanation for this boundary was given by Brassler & Lehto (2002), who investigated the role of secular resonances in the motion of Trojans of the terrestrial planets. In the case of Venus Trojans, the ν_3 (Earth) and ν_4 (Mars) secular resonances act in the region $17^\circ - 30^\circ$.

We present in this paper a stability analysis over gigayear timescales. In order to investigate the long-term stability of Venus Trojans, we apply Laskar's frequency map analysis (FMA), as in all our previous studies of Trojan dynamics carried out in the frame of the MATROS project. The FMA method (Laskar et al. 1992; Laskar 1993a, 1993b) determines the regions in phase space in which the most stable orbits are located. It also yields values for the frequencies of the intrinsic variables of the problem. Moreover, the FMA allows us to derive semiempirical expressions for the secular frequencies of Trojan motion as a function of proper orbital elements. The presence and role of secular resonances for instability can then be easily investigated.

2. THE SEARCH FOR THE MOST STABLE VENUS TROJANS

The FMA method is applied to the orbits of Venus Trojans perturbed by all planets except Pluto. The fundamental frequencies of the nonsingular variables h and k , as well as of p and q , play a key role. The variables are defined, as usual, by

$$\begin{aligned} h &= e \cos(\tilde{\omega}), \\ k &= e \sin(\tilde{\omega}), \\ p &= i \cos(\Omega), \\ q &= i \sin(\Omega), \end{aligned}$$

where e , i , $\tilde{\omega}$, and Ω refer to eccentricity, inclination, longitude of perihelion, and longitude of ascending node, respectively. We describe the method for h and k . It applies similarly to p and q and eventually to other fundamental frequencies. The numerical integration of a Venus Trojan orbit yields discrete values h_j and k_j at different instants of time t_j . These discrete values are combined in a complex time series $h_j + ik_j$, which is generally quasi-periodic. The corresponding Fourier spectrum contains the main frequencies of the solar system and, most important for the analysis, the proper orbital frequency g , which is also called the "fundamental frequency." Its amplitude is the proper eccentricity of the orbit. The change of the fundamental frequency g between time windows or, synonymously, the diffusion rate of g is a measure for chaos. A large diffusion rate means a strongly chaotic orbit.

Using the FMA method, we construct a synthetic filtered time series for time windows that contains only the major frequencies

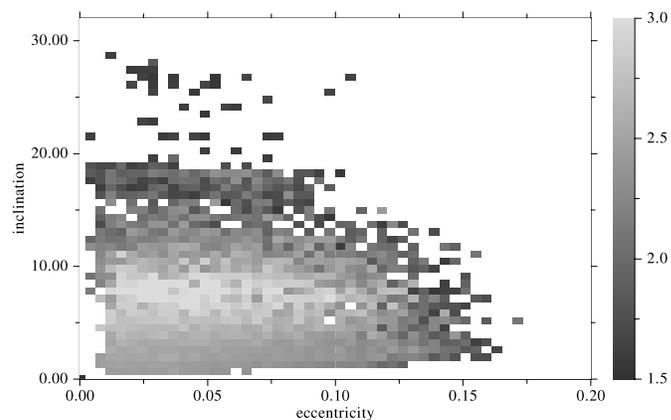


FIG. 1.—Diffusion map of Venus Trojans in the e_p - i_p plane. The stable region is located at inclinations lower than $\sim 12^\circ$. [See the electronic edition of the Journal for a color version of this figure.]

but models the original signal very closely. Since the frequencies are usually not orthogonal, a Gramm-Schmidt orthogonalization is applied, which determines step-by-step frequencies and amplitudes, correcting at each step amplitudes of previously determined frequencies.

In the case of a nonchaotic orbit, the fundamental frequency g is constant, while it diffuses in the case of chaotic motion. The higher the frequency diffusion rate, the more chaotic the orbit. In practice, we do not directly measure the diffusion rate, but we calculate an equivalent parameter: the dispersion σ of g . We divide the whole integration interval into overlapping windows of fixed length and consider the time series $h + ik$ over a window. The window length should be at least 10 times as wide as the period of g . For each window w , we determine the fundamental frequency g_w . Denoting by s_g the standard deviation of g_w over all windows, the dispersion σ_g is defined by $\sigma_g = -\log(s_g/g)$.

In order to perform the FMA analysis for Venus Trojans, we integrate 7000 orbits over 5 Myr using the WHM integrator (Wisdom & Holman 1991), which is part of the SWIFT package (Levison & Duncan 1994). A fixed time step of 2.5 days is used. Starting orbital elements are produced by a random generator. Orbits unstable over 5 Myr are rejected. The time windows have a length of 1 Myr. They overlap for 0.5 Myr.

The proper eccentricity e_p and proper inclination i_p , which are the amplitudes of the corresponding fundamental frequencies in the $h + ik$ and $p + iq$ time series, are used to classify each Trojan orbit in our sample. Of particular interest is the proper eccentricity, since it affects the orbital eccentricity, which is an important parameter for the stability of a Trojan orbit. It controls possible approaches to Venus or to the Earth. Besides the proper eccentricity, the libration amplitude D of the tadpole orbit is another important parameter. In the case of a large amplitude, the Trojan may be ejected out of the Trojan region after a close approach with Venus. We therefore present the dispersion σ_g , which is equivalent to the diffusion rate of the fundamental frequency g , in two-dimensional diagrams using either e_p , i_p , or D as variables. The diagrams are called diffusion maps.

2.1. Diffusion Maps and Location of the Dynamically Most Stable Regions

Figure 1 is a diffusion map for Venus Trojan orbits in the (e_p, i_p) space. Large σ values mean slow diffusion and represent the most stable orbits. For values below a certain threshold, an orbit

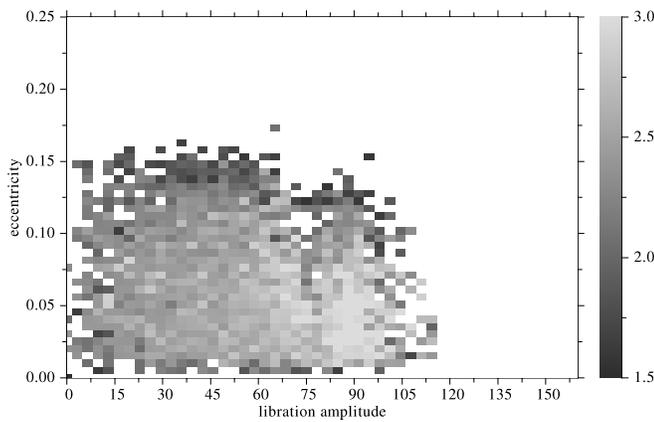


FIG. 2.—Diffusion map of Venus Trojans in the D - e_p plane. The stable region is located at large libration amplitudes for $D > 60^\circ$. Only a few isolated stable orbits can be found at low libration amplitudes. [See the electronic edition of the *Journal* for a color version of this figure.]

may be destabilized so quickly that it is not possible to compute its proper elements correctly. In our case, we set this threshold to 1.55. The most stable orbits are clearly concentrated in a small region with proper inclination between 5° and 10° and proper eccentricity lower than ~ 0.13 . Figure 2 shows, for orbits with proper inclinations less than 15° , the dispersion σ with respect to proper libration amplitude D . It is interesting to note that the most stable orbits have the largest libration amplitudes, ranging from 60° to 110° . This region is significantly smaller than the one outlined by Tabachnik & Evans (2000) with their 75 Myr survey. The same result was obtained for Trojan orbits of Saturn (Marzari et al. 2002), Uranus, and Neptune (Marzari et al. 2003b). Also, the most stable Mars Trojans may have very large libration amplitudes. This must be due to the perturbations of the other planets, since the most stable orbits in the three-body problem are near the Lagrangian points. Only for Jupiter Trojans do the most stable orbits have small libration amplitudes (Marzari et al. 2003a).

The diffusion maps are the first step to understanding the long-term dynamics of Venus Trojans, since they show relative timescales for stable motion. What is still needed is to interpret the relative timescale in years and, in addition, to find the mechanisms that destabilize the orbits, in particular at higher eccentricities and inclinations.

3. LONG-TERM DYNAMICS OF VENUS TROJANS

The diffusion maps allow us to identify the Trojan orbits that evolve in the phase space at a lower rate. The FMA indicates the orbits that would survive longer as Trojans, but it does not give an absolute timescale. In order to estimate the dynamical lifetimes of the most stable Trojans in years, we numerically integrate a population of Trojans with the lowest diffusion rate according to Figures 1 and 2. The perturbations of all planets, except Pluto, are taken into account.

The decay of this population is shown in Figure 3. After about 6×10^8 yr, half of the population is destabilized. The decay rate is nearly linear. The whole population disappears after 1.2 Gyr. We therefore conclude that it is very unlikely that primordial Venus Trojans will be detected. We do not say that there are currently no small bodies on tadpole orbits since, as outlined above, there may be transient Venus Trojans.

The Yarkovsky effect may further reduce the dynamical lifetime of Venus Trojans. The dashed curve in Figure 3 shows the decay of the population when subjected to the Yarkovsky effect.

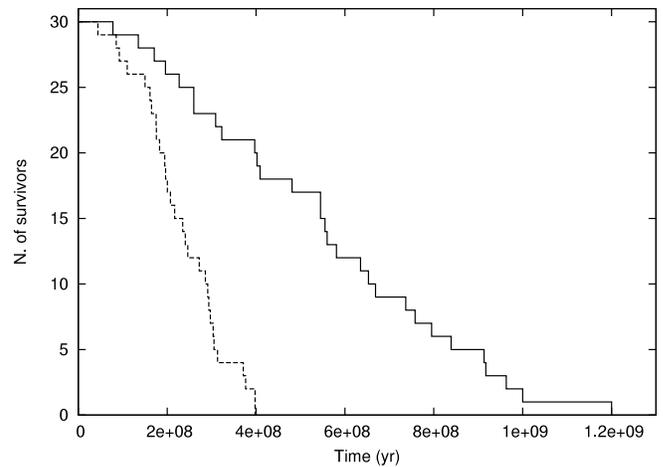


FIG. 3.—Survival curve for Venus Trojans. The solid line represents the outcome of an integration without the Yarkovsky effect. The dashed line shows the survival curve for 1.5 km Trojans perturbed by the Yarkovsky force. [See the electronic edition of the *Journal* for a color version of this figure.]

We use the *swift-rms3* integrator modified by M. Broz to include the Yarkovsky effect.¹ The physical parameters for the Yarkovsky force are similar to those used for Mars Trojans by Scholl et al. (2005). The bulk density is set to 2.5 g cm^{-3} , the surface density to 1.5 g cm^{-3} , the surface conductivity to $K = 0.001 \text{ W m}^{-1} \text{ K}^{-1}$, and the emissivity to 0.9. The rotation rate is assumed to be 6 hr and the diameter 1.5 km, and a large albedo equal to 0.42, derived for the two Mars Trojans by Rivkin et al. (2003), is adopted. The Yarkovsky force reduces the half-life of Venus Trojans to $\sim 2.5 \times 10^8$ yr, and after 4×10^8 yr all the bodies escape.

4. SYNTHETIC SECULAR THEORY

In order to understand the shape of the stable island in the phase space, we have to compare the proper frequencies of Trojan orbits with the fundamental frequencies of the solar system. In this way, we can map the location of the secular resonances that affect the motion of Venus Trojans. We use the standard approach pioneered by Milani (1994), in which a least-squares fit of the g and s frequencies is performed. These frequencies, i.e., the circulation period of the perihelion longitude and of the node, are represented by polynomials in the proper orbital elements, as follows:

$$g = 29.052 + 0.137x^2 - 50.216y^2 + 3.602z^2 - 11.734x^2y^2 - 0.994x^2z^2 - 2.762y^2z^2 - 0.669x^4 + 63.764y^4 - 0.777z^4, \quad (1)$$

$$s = -12.155 - 2.015x^2 + 19.740y^2 + -1.298z^2 + 12.305x^2y^2 - 0.274x^2z^2 + 0.368y^2z^2 - 1.316x^4 - 19.293y^4 + 0.162z^4. \quad (2)$$

The proper elements are rescaled in the following way:

$$x = \frac{e_p}{0.15}, \quad y = \frac{\sin I_p}{0.6}, \quad z = \frac{da}{0.001 \text{ AU}}. \quad (3)$$

¹ See <http://sirrah.troja.mff.cuni.cz/~mira/mp>.

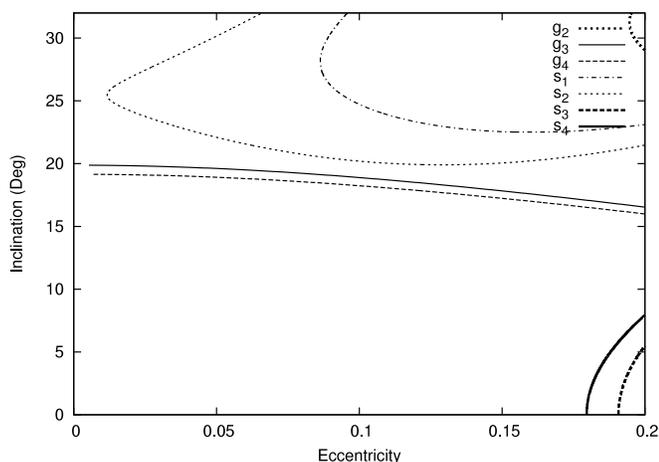


FIG. 4.—Location of the main secular resonances perturbing the motion of putative Venus Trojans. [See the electronic edition of the *Journal* for a color version of this figure.]

The amplitude of the semimajor axis oscillation da is related to the proper libration amplitude D through the formula $D = da/0.0019597$ derived from the analytical theory of Erdi (1988). All the frequencies are expressed in arcseconds per year, and the relative error in each coefficient is less than 1%.

In Figure 4 we plot the location of the major secular resonances crossing the Trojan region. By comparing Figure 1 with Figure 4 it is evident that the secular resonances with the fundamental frequencies g_2 , g_3 , and g_4 are responsible for the upper limit of the stable island in inclination.

5. DISCUSSION AND CONCLUSIONS

As for Mars, some kilometer-sized asteroids may be lurking in the Lagrangian points of Venus. However, they are hard to spot

from ground-based observations because of their small solar elongations, and the few surveys have not yet found any. Numerical studies of the long-term stability of putative Venus Trojans (Mikkola & Innanen 1992; Tabachnik & Evans 2000) are limited in time and cover a period of up to 100 Myr. This time span is not long enough to answer the critical question of whether bodies trapped at the Lagrangian points of Venus in the early phases of the solar system evolution could have survived until the present. If Venus Trojans are found in the future, we want to have clues to their origin, either primordial or due to a more recent capture. To investigate the long-term stability of Venus Trojans we have used the frequency map analysis, as in all the previous studies of Trojan dynamics related to the MATROS project. This method allows us to outline the region in phase space in which the most stable orbits are located. We find that for Venus Trojans this region is limited in the phase space and is significantly smaller than that outlined by Tabachnik & Evans (2000) with their 75 Myr survey. Long-term integrations within this region allow us to set an upper limit to the lifetime of Trojan orbits that is much shorter than the solar system age. As a consequence, any population of primordial Trojans would have disappeared by now. The only primordial planetesimals formed in the terrestrial planetary region appear so far to be the Mars Trojans. Both Mars and Venus Trojan orbits are chaotic, but the diffusion speed for Mars Trojans is significantly lower. Moreover, the Yarkovsky effect is more efficient for Venus Trojans as compared to Mars Trojans, since they are closer to the Sun.

We cannot exclude that future surveys may detect transient Venus Trojans, since numerical integrations of known Venus co-orbital asteroids show how these bodies can become temporary Trojans. However, these are near-Earth asteroids delivered into the terrestrial region from the asteroid belt. An investigation of the stability of Earth Trojan orbits is under way and will be the subject of a forthcoming paper.

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